Candidate surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number | Candidate Number

Wednesday 5 June 2019

Morning (Time: 2 hours) | Paper Reference **9MA0/01**

Mathematics

Advanced
Paper 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P58353A

1/1/1/1/1/1/C2/C2/

P 5 8 3 5 3 A 0 1 4 4

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

when $f(k) = 0$ then $(x-k)$ is a factor of $f(x)$

$f(-3) = 0$ $(x - -3)$

$f(-3) = 3(-3)^3 + 2a(-3)^2 - 4(-3) + 5a = 0$ 1

$3(-27) + 2a(9) + 12 + 5a = 0$

$-81 + 18a + 12 + 5a = 0$

$-69 + 23a = 0$ $\therefore a = 3$ 1

$+69$ $+69$

$23a = 69$ 1 $a = 69/23 = 3$

$\div 23$ $\div 23$

P 5 8 3 5 3 A 0 2 4 4

2.

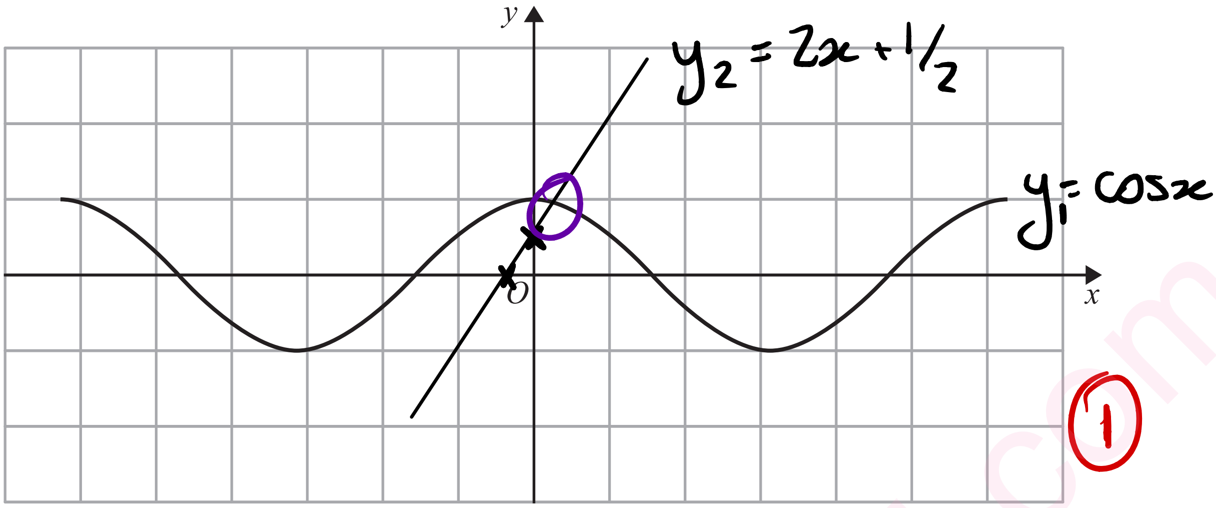


Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

(a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

$\cos x = 2x + 1/2$

$y_1 = \cos x$

$y_2 = 2x + 1/2$

Given that the root of the equation is α, and that α is small,

(b) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

a) $y_2 = 0$

$0 = 2x + 1/2$

$2x = -1/2$

$x = -1/4$

Since there is only one point of intersection between the functions $y_1 = \cos x$ and $y_2 = 2x + 1/2$ it follows the equation $\cos x - 2x - 1/2 = 0$ has only one real root

1

Question 2 continued

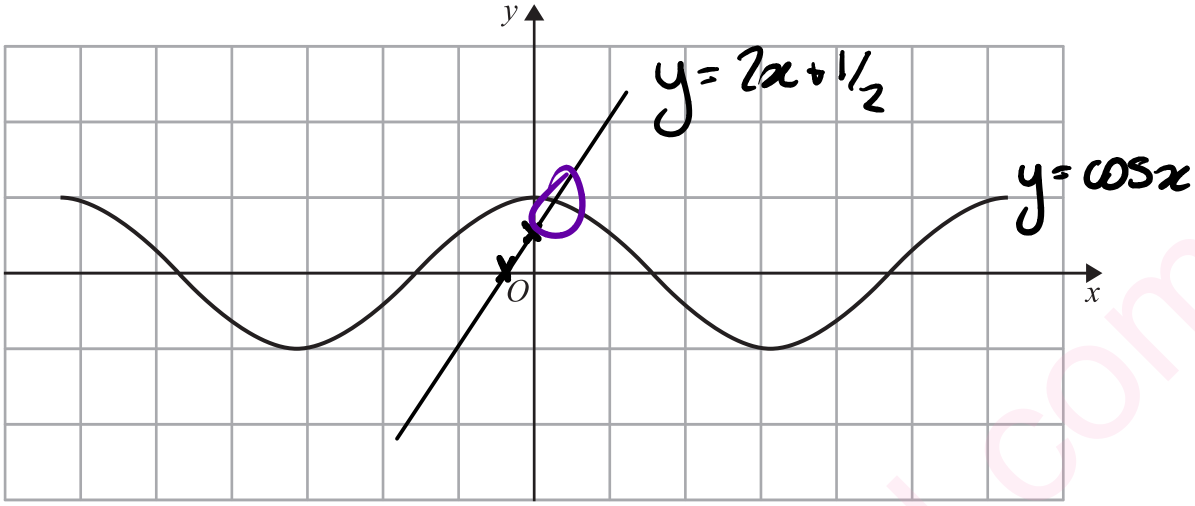


Diagram 1

b) $\cos x = 1 - \frac{1}{2}x^2$

Sub into 'original equation'

$1 - \frac{1}{2}x^2 - 2x - \frac{1}{2} = 0$ ①

$-\frac{1}{2}x^2 - 2x + \frac{1}{2} = 0$

$\times -2$

$x^2 + 4x - 1 = 0$ ①

$x = -2 + \sqrt{5}$

$x = -2 - \sqrt{5}$ x reject

$\therefore \alpha \approx 0.236$ ①

(Total for Question 2 is 5 marks)

3. differentiate $y = \dfrac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$

(a) Show that $\dfrac{dy}{dx} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(b) Hence deduce the range of values for x for which $\dfrac{dy}{dx} < 0$

(1)

a)

$f(x) = 5x^2 + 10x \qquad f'(x) = 10x + 10$

$g(x) = (x+1)^2 \qquad g'(x) = 2(x+1)$ 1

$2(1)(x+1)^1$

$= 2(x+1)$

$dy/dx = \dfrac{(10x+10)(x+1)^{\cancel{2}\,1} - 2(5x^2+10x)(x+1)}{(x+1)^{\cancel{4}\,3}}$ 1

$= \dfrac{(10x+10)(x+1) - 2(5x^2+10x)}{(x+1)^3}$ 1

$= \dfrac{\cancel{10x^2} + \cancel{10x} + \cancel{10x} + 10 - \cancel{10x^2} - \cancel{20x}}{(x+1)^3}$

$= \dfrac{10}{(x+1)^3}$ $\qquad A = 10$, $n = 3$

1

P 5 8 3 5 3 A

Question 3 continued

b) $\frac{10}{(x+1)^3} < 0$

Possitive

< 0 essentially means negative

denominator must be negative

$\frac{+}{+} = + > 0$

$\frac{+}{-} = - < 0$

$(x+1)^3 < 0$

$x+1$ has to be negative

$(+)^3 = +$

$(-)^3 = -$

$x + 1 < 0$

$-1 \quad -1$

$x < -1$ 1

(Total for Question 3 is 5 marks)

P 5 8 3 5 3 A 0 7 4 4

4. (a) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$(4-x)^{-\frac{1}{2}} \quad \frac{1}{\sqrt{4-x}}$$

$a^{-1} = \frac{1}{a}$ $\quad a^{1/2} = \sqrt{a}$

$a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$
Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\frac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

snippet from formula booklet

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$ 1

a) $(4-x)^{-\frac{1}{2}} = [(4-x)]^{-\frac{1}{2}} = [4(1-\frac{x}{4})]^{-\frac{1}{2}}$

$= \frac{1}{2}(1-\frac{x}{4})^{-\frac{1}{2}}$ 1

$\frac{1}{2}(1+-\frac{x}{4})^{-\frac{1}{2}} = \frac{1}{2}\left[1 + (\frac{-1}{2})(\frac{-x}{4}) + \frac{(\frac{-1}{2})(\frac{-3}{2})}{2}\left(\frac{-x}{4}\right)^2 + \ldots\right]$ 1

$= \frac{1}{2}\left[1 + \frac{x}{8} + \frac{3x^2}{128} + \ldots\right]$

$\frac{1}{\sqrt{4-x}} = \frac{1}{2} + \frac{x}{16} + \frac{3x^2}{256} + \ldots$ 1

P 5 8 3 5 3 A 0 8 4 4

Question 4 continued

snippet from formula booklet

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1\times 2}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{1\times 2\times \ldots \times r}x^r + \ldots \quad (|x| < 1, n \in \mathbb{R})$$

b) i) $\frac{1}{2}\left(1 + -\frac{x}{4}\right)^{-\frac{1}{2}}$ $\quad \left|\frac{-x}{4}\right| < 1 \quad |-x| < 4$

$\therefore |x| < 4$ (1)

$|-14| = 14 \quad 14 > 4$ which means $x = -14$ should not be used since $x = -14$ is not valid for $|x| < 4$

b) ii) $x = \frac{-1}{2}$ because it is closest to zero (1)

(Total for Question 4 is 6 marks)

Turn over

P 5 8 3 5 3 A 0 9 4 4

5.

$$f(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(a) Write f(x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.
(3)

(b) Sketch the curve with equation $y = f(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.
(3)

(c) (i) Describe fully the transformation that maps the curve with equation $y = f(x)$ onto the curve with equation $y = g(x)$ where

$$g(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$
(4)

a) $f(x) = 2x^2 + 4x + 9$

$= 2(x^2 + 2x) + 9$ 1

$= 2[(x+1)^2 - 1] + 9$ 1

$= 2(x+1)^2 - 2 + 9$

$= 2(x+1)^2 + 7$ 1

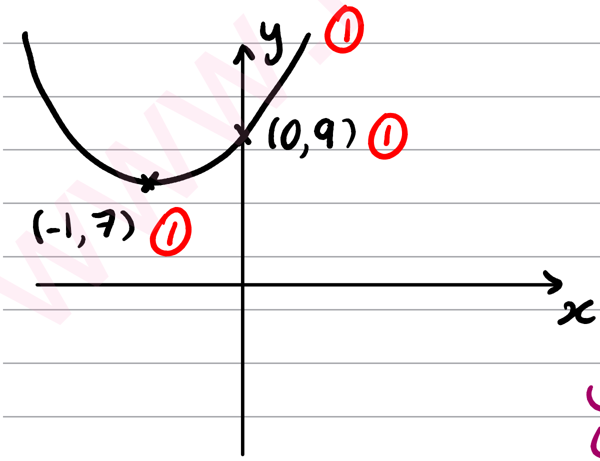
b) $y = 2(x+1)^2 + 7$

y intercept when $x = 0$

$y = 2(0+1)^2 + 7$
$= 2 + 7$
$= 9$ $\therefore$ y intercept $(0,9)$

x intercept when $y = 0$

$0 = 2(x+1)^2 + 7$
$2(x+1)^2 = -7$
$(x+1)^2 = \frac{-7}{2}$ ✓ curve doesn't intercept x axis (no real roots)



Turning Point

$y = (x-a)^2 + b$ turning point (a,b)

$y = 2(x - -1)^2 + 7$ $\therefore$ turning point $(-1, 7)$

Question 5 continued

c) i) $f(x) = 2x^2 + 4x + 9$

$g(x) = 2(x-2)^2 + 4x - 3$

$f(x-a)$ translation by vector $\begin{pmatrix} a \\ 0 \end{pmatrix}$

$f(x-2) = 2(x-2)^2 + 4(x-2) + 9$

$= 2(x-2)^2 + 4x - 8 + 9$

$= 2(x-2)^2 + 4x + 1$

$f(x) + b$ translation by vector $\begin{pmatrix} 0 \\ b \end{pmatrix}$

$f(x-2) - 4 = 2(x-2)^2 + 4(x-2) + 9 - 4$

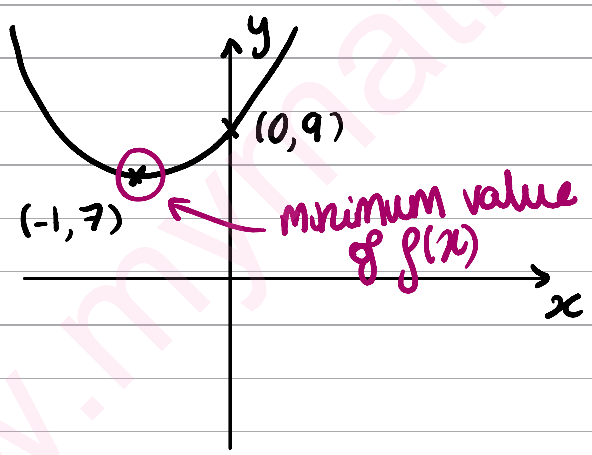
$= 2(x-2)^2 + 4x - 8 + 9 - 4$

$= 2(x-2)^2 + 4x - 3$

$g(x) = f(x - \underset{a}{2}) \underset{b}{- 4}$

$\therefore$ The transformation that maps $y = f(x)$ onto $y = g(x)$ is a translation by vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ (2)

c) ii)



$h(x) = \dfrac{21}{2x^2 + 4x + 9}$

$= \dfrac{21}{f(x)}$

$f(x) \to \pm\infty$

$h(x) \to 0$

Maximum $h(x)$ when $f(x)$ is at its minimum

$\dfrac{21}{7} = 3$ (1)

$0 < h(x) \leq 3$ (1)

6. (a) Solve, for $-180° \leqslant \theta \leqslant 180°$, the equation

$$5\sin 2\theta = 9\tan\theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(b) Deduce the smallest positive solution to the equation

$$5\sin(2x - 50°) = 9\tan(x - 25°)$$

(2)

a) $5\sin 2\theta = 9\tan\theta$

using $\sin 2\theta = 2\sin\theta\cos\theta$

$5(2\sin\theta\cos\theta) = 9\tan\theta$

using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$10\sin\theta\cos\theta = \dfrac{9\sin\theta}{\cos\theta}$ (1)

$10\sin\theta\cos^2\theta = 9\sin\theta$

$10\sin\theta\cos^2\theta - 9\sin\theta = 0$

$\sin\theta(10\cos^2\theta - 9) = 0$

$\sin\theta = 0$ OR $10\cos^2\theta - 9 = 0$ (1)

$\theta = \sin^{-1}(0) = 0°$

From calculator

$10\cos^2\theta = 9$

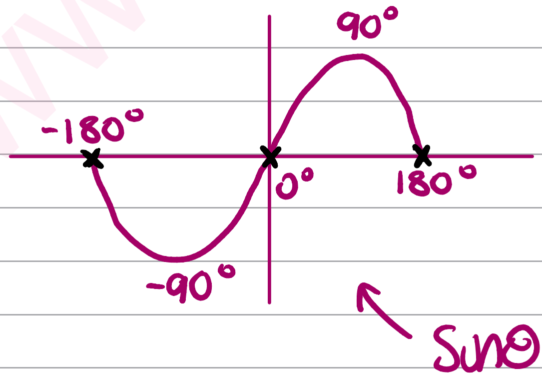
$\cos^2\theta = \dfrac{9}{10}$

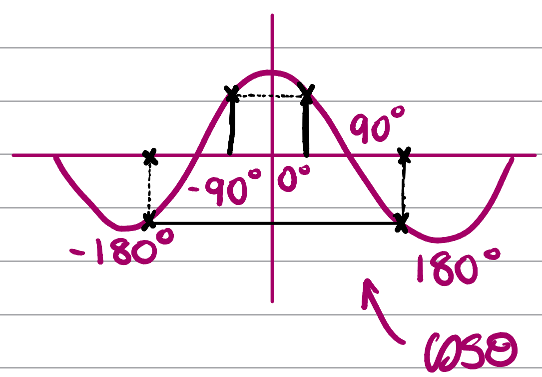
$\cos\theta = \dfrac{\pm 3}{\sqrt{10}}$

From Calculator

$\theta = \cos^{-1}\left(\dfrac{3}{\sqrt{10}}\right) = 18.4°$ (1)

$\theta = \cos^{-1}\left(\dfrac{-3}{\sqrt{10}}\right) = 161.6°$





Question 6 continued

a) Solutions found by using $\cos\theta$ and $\sin\theta$ curves

$\theta = 0°, 180°, -180°, 18.4°, -18.4°,$
$161.6°, -161.6°$ 3

b) $\theta = x - 25°$
$x = \theta + 25°$

sub $\theta = -18.4°$

$x = -18.4° + 25°$ 1
$= 6.6°$ 1

7. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000 ↗ $t=0$
- its value after one year is £16 000 → $t=1$

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

a) $V = Ae^{kt}$ ↙ general exponential model 1

When car is new: $20000 = Ae^{k(0)}$ 1

$20000 = A(1) \quad \therefore A = 20000$

$V = 20000e^{kt}$

After one year: $16000 = 20000e^{k(1)}$

$16000 = 20000e^{k}$ 1

$e^{k} = \frac{4}{5}$

$\ln e^{k} = \ln\frac{4}{5}$

$k = \ln\frac{4}{5} = -0.223$

$V = 20000e^{-0.223t}$ 1

Question 7 continued

b) $V = 20\,000e^{-0.223(10)}$
$= £2150$ (1)

Actual value of car A after 10 years is £2000

£2150 ≈ £2000 ∴ Our model is reliable (1)

c) $V = Ae^{kt}$

'A' value will be the
same for car B since
car A and B have same
value when ∴ we have to ajust 'k' value

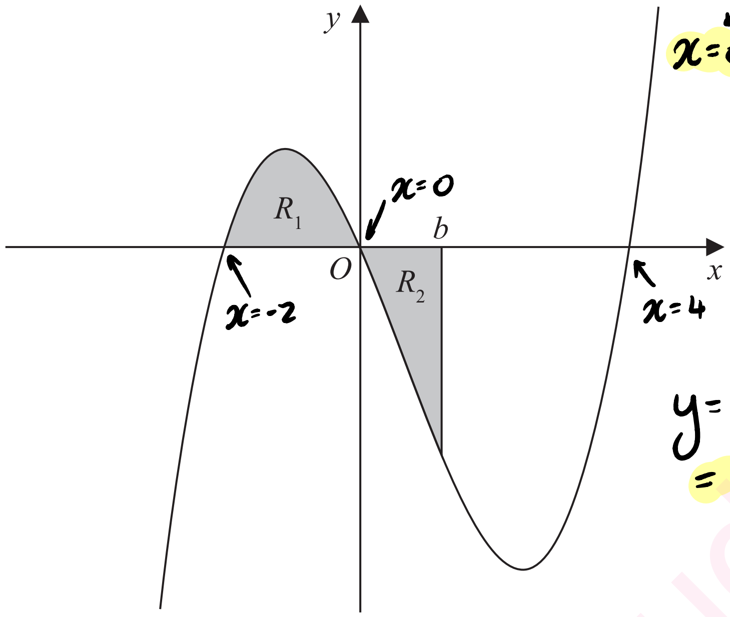
Make "-0.223" (the 'k' value) less negative (1)

8.

$0 = x(x+2)(x-4)$

$x=0$ $\quad x+2=0$ $\quad x-4=0$

$x=-2$ $\quad x=4$



$y = x(x^2-2x-8)$

$= x^3-2x^2-8x$ (1)

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x+2)(x-4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(a) Show that the exact area of R_1 is $\frac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(b) verify that b satisfies the equation

$$(b+2)^2(3b^2-20b+20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(c) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

a) $\int_{-2}^{0} x^3-2x^2-8x \, dx$ (1)

$= \left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_{-2}^{0}$ (1) *

* $= 0 - \left[\frac{1}{4}(-2)^4 - \frac{2}{3}(-2)^3 - 4(-2)^2\right]$

$= -\left[-\frac{20}{3}\right]$ (1)

$= \frac{20}{3}$ as needed

P 5 8 3 5 3 A 0 2 2 4 4

Question 8 continued

b) $\int_0^b x^3 - 2x^2 - 8x \, dx = \frac{-20}{3}$ negative since region is below the x axis

$\left[\frac{1}{4}x^4 - \frac{2}{3}x^3 - 4x^2\right]_0^b = \frac{-20}{3}$

$\frac{1}{4}(b)^4 - \frac{2}{3}(b)^3 - 4(b)^2 - 0 = \frac{-20}{3}$

$(b+2)^2(3b^2 - 20b + 20) = 0$

$(b^2 + 4b + 4)(3b^2 - 20b + 20) = 0$

$3b^4 - 20b^3 + 20b^2 + 12b^3 - 80b^2$

$+ \cancel{80b} + 12b^2 - \cancel{80b} + 80 = 0$ 1

$3b^4 - 8b^3 - 48b^2 + 80 = 0$

equation B

$\frac{b^4}{4} - \frac{2b^3}{3} - 4b^2 = \frac{-20}{3}$ 1 $\times 3$

$\frac{3b^4}{4} - 2b^3 - 12b^2 = -20$ $\times 4$

$3b^4 - 8b^3 - 48b^2 = -80$

$3b^4 - 8b^3 - 48b^2 + 80 = 0$ 1

equation A

Since equation A and B are identical we have verified that b satisfies the equation $(b+2)^2(3b^2 - 20b + 20) = 0$

1

P 5 8 3 5 3 A 0 2 3 4 4

Question 8 continued

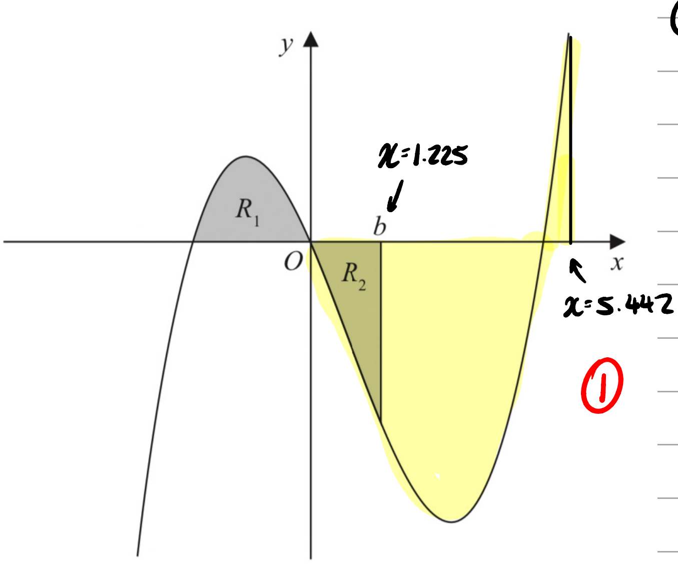


Figure 2

c)

$$\int_0^b y\,dx = \frac{-20}{3}$$

$$\int_0^{1.225} y\,dx = \frac{-20}{3}$$

$$\int_0^{5.442} y\,dx = \frac{-20}{3}$$

The net area between 0 and 5.442 is $\frac{-20}{3}$
(The yellow shaded region) (1)

9. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(a) show that

$$a = \frac{b^2}{b-1}$$

(3)

(b) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

a) $\log a - \log b = \log(a-b)$ ①

using $\log a - \log b = \log\left(\frac{a}{b}\right)$

$\log\left(\frac{a}{b}\right) = \log(a-b)$

$\therefore \frac{a}{b} = a - b$

$\times b$

$a = ab - b^2$

$+ b^2$ ①

$b^2 + a = ab$

$- a$

$b^2 = ab - a$

$b^2 = a(b-1)$

$\div (b-1)$

$a = \frac{b^2}{b-1}$ as needed ①

b) $a = \frac{b^2}{b-1}$

$b - 1 \neq 0 \therefore b \neq 1$

Since $a > 0$ we know

Always possitive

$\frac{b^2}{b-1} > 0$ ①

$\therefore b-1$ must be possitive because $\frac{+}{-} < 0$

So $b - 1 > 0$

$\therefore b > 1$ ①

Prog by exhaustion

10. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

1, 2, 3, 4, ...

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

i) For when n is even
let $n = 2k$

1

$n^2 + 2 = (2k)^2 + 2$
$= 4k^2 + 2$

divisible by 4

So when n is even n^2+2 is not divisible by 4 since it's 2 more than a multiple of 4 and 2 isn't divisible by 4

1

For when n is odd
let $n = 2k+1$

1

$n^2 + 2 = (2k+1)^2 + 2$
$= 4k^2 + 4k + 1 + 2$
$= 4(k^2 + 4) + 3$

divisible by 4

So when n is odd n^2+2 is not divisible by 4 since it's 3 more than a multiple of 4 and 3 isn't divisible by 4

1

$\therefore$ because n is not divisible by 4 when n is odd and when n is even this means that n^2+2 is not divisible by 4 for $n \in \mathbb{N}$

Proof by exhaustion

(i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

$1, 2, 3, 4, \ldots$

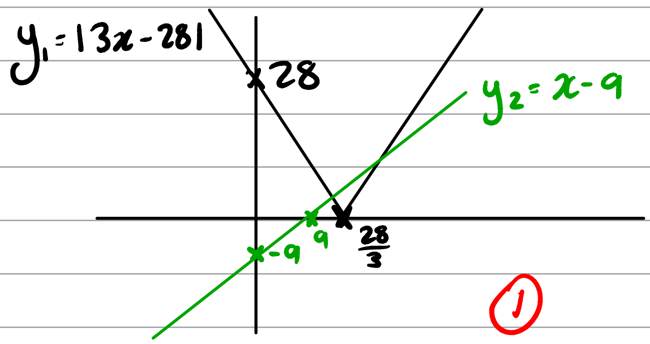
(4)

$1, 3.5, \sqrt{5}$

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."
State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

ii) $|3x - 28| \geq x - 9$



1

$y_1 = |3x - 28|$

$0 = 3x - 28$

$3x = 28$

$x = \frac{28}{3}$

$y_2 = x - 9$

$0 = x - 9$

$x = 9$

Working out x intercepts

$\therefore$ The statement is sometimes true, since there are points where $|3x - 28| < x - 9$ and points $|3x - 28| > x - 9$

1

(Total for Question 10 is 6 marks)

P 5 8 3 5 3 A 0 2 9 4 4

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre. After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, **(2)**

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race. **(4)**

a) Total time for first 6 km $= (6 \times 4) + (6 \times 1.05) + (6 \times 1.05^2)$ ①

minutes km

$= 36.915$ minutes $= 36$ minutes 55 seconds ①

$(60 \times 0.915 = 54.9 \approx 55)$

b) 5^{th} km : 6×1.05^1 $5-1=4$

6^{th} km : 6×1.05^2 $6-2=4$

7^{th} km : 6×1.05^3 $7-3=4$ ①

$\therefore$ it follows that the time for r^{th} km is $6 \times 1.05^{r-4}$

c) total time $= 24$ minutes $+ \sum_{r=5}^{r=20} 6 \times 1.05^{r-4}$ ① ①

4km

Geometric Series

$a = 6.3$

$r = 1.05$

$n = 16$

$S_n = \dfrac{a(1-r^n)}{1-r}$

$= 24$ minutes $+ \dfrac{6.3(1-1.05^{16})}{1-1.05}$ ①

$= 173.042$ minutes $= 173$ minutes 3 seconds ①

(60×0.042)

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

P 5 8 3 5 3 A 0 3 0 4 4

12. $$f(x) = 10e^{-0.25x}\sin x, \qquad x \geqslant 0$$

(a) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

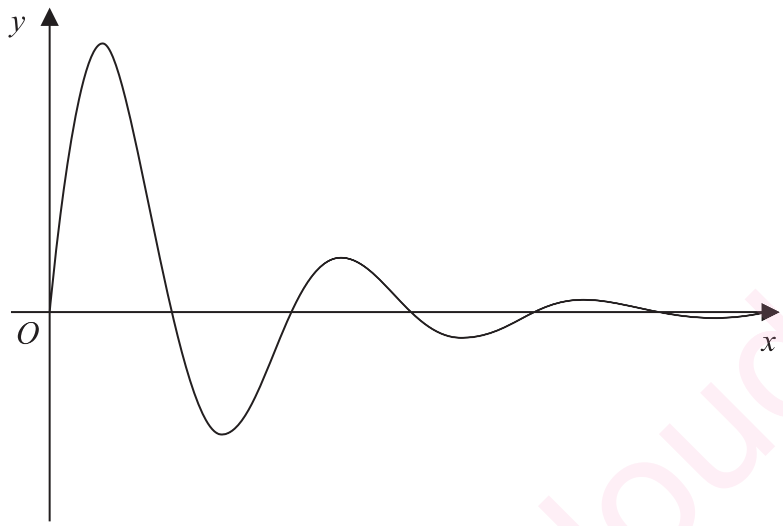


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(b) Sketch the graph of H against t where

$\nearrow f(x)$

$$H(t) = \left|10e^{-0.25t}\sin t\right| \qquad t \geqslant 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(c) the maximum height of the ball above the ground between the first and second bounce.

(3)

(d) Explain why this model should not be used to predict the time of each bounce.

(1)

Question 12 continued

a) $f(x) = 10e^{-0.25x}\sin x$

$f'(x) = -0.25(10e^{-0.25x})\sin x + \cos x(10e^{-0.25x})$

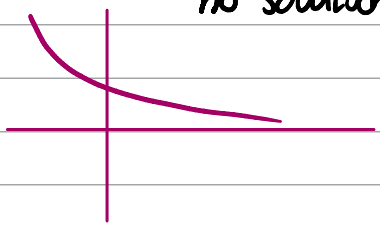
$= -2.5\sin x e^{-0.25x} + 10\cos x e^{-0.25x}$ (2)

$0 = -2.5\sin x e^{-0.25x} + 10\cos x e^{-0.25x}$

$= e^{-0.25x}(-2.5\sin x + 10\cos x)$ (1)

$\therefore e^{-0.25x} = 0$ or $-2.5\sin x + 10\cos x = 0$

reject because no solution



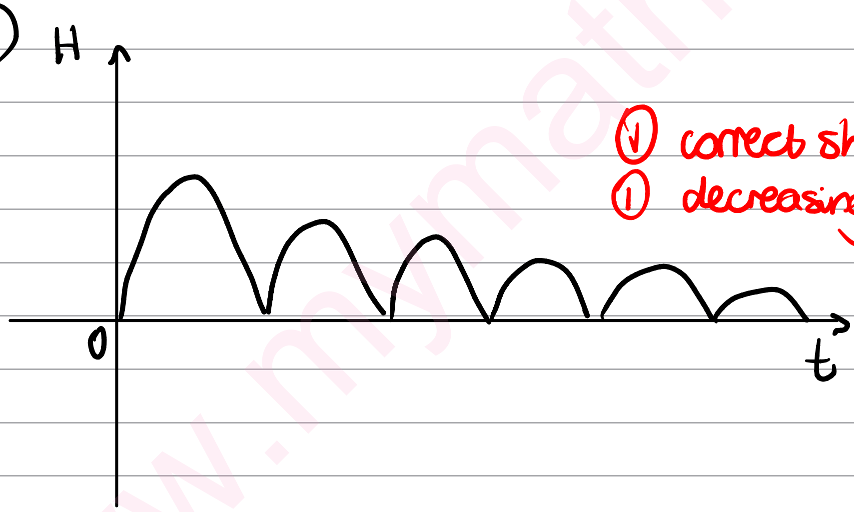
$10\cos x = 2.5\sin x$

$\frac{10}{2.5} = \frac{\sin x}{\cos x}$ (1)

$\frac{\sin x}{\cos x} = \tan x$

$\tan x = 4$ as needed

b)

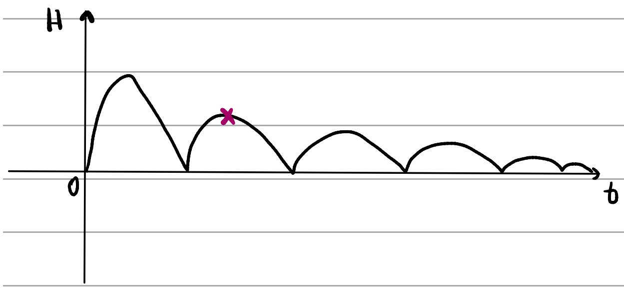


(1) correct shape

(1) decreasing heights

Question 12 continued

c)



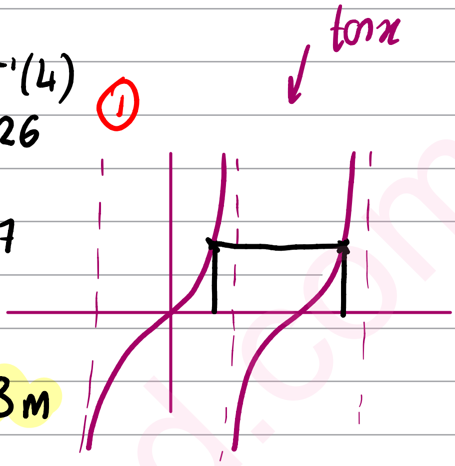
$\tan x = 4$

$x = \tan^{-1}(4)$

$= 1.326$ (1)

$+\pi$

$= 4.47$



(1)

$H(4.47) = |10e^{-0.25(4.47)} \times \sin(4.47)| = 3.18\text{m}$

(1)

d)

The times between each bounce should not stay the same when the heights of each bounce is getting smaller (1)

(Total for Question 12 is 10 marks)

P 5 8 3 5 3 A 0 3 7 4 4

13. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \frac{1}{2}\right)$ and has two vertical asymptotes with equations $x = 2$ and $x = -3$

(a) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

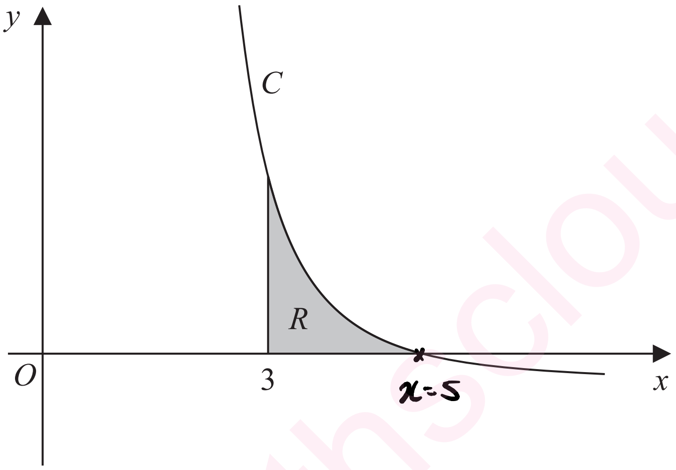


Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(b) Show that the exact value of the area of R is $a\ln 2 + b\ln 3$, where a and b are rational constants to be found.

(8)

a i) Verticle asymptotes when $(2x-q)(x+3)=0$

$2x-q=0 \qquad x+3=0$

$2x=q \qquad x=-3$

1 $\quad x=2$

$2(2)=q$

$\therefore q=4$ as needed

a ii) $y=\dfrac{p-3x}{(2x-4)(x+3)}$ $\qquad \frac{1}{2}=\dfrac{p-9}{12}$

$\frac{1}{2}=\dfrac{p-3(3)}{(2(3)-4)(3+3)}=\dfrac{p-9}{12}$ $\qquad 6=p-9$ 1

$2\times 6=12$ 1 $(3,\frac{1}{2})$ $\qquad p=6+9$

$\therefore p=15$ as needed

Question 13 continued

b) $y = \frac{15-3x}{(2x-4)(x+3)}$

$0 = \frac{15-3x}{(2x-4)(x+3)}$

$0 = 15-3x$

$3x = 15$

$x = 5$

$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ ②

$15-3x = A(x+3) + B(2x-4)$

$2x-4=0 \Rightarrow x=2$

let $x=-3$ $24 = B(-10)$ $\therefore B = -2.4$

let $x=2$ $9 = 5A$ $\therefore A = 1.8$

$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ ①

$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

$\int_3^5 \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}\,dx = \Big[0.9\ln|2x-4| - 2.4\ln|x+3|\Big]_3^5$

$f(x) = 2x-4$ $f'(x) = 2$ $\frac{1.8}{2} = 0.9$

$f(x) = x+3$ $f'(x) = 1$ $\frac{2.4}{1} = 2.4$

④

$= 0.9\ln|2(5)-4| - 2.4\ln|5+3| - \Big[0.9\ln|2(3)-4| - 2.4\ln|3+3|\Big]$

$= 0.9\ln|6| - 2.4\ln|8| - 0.9\ln|2| + 2.4\ln|6|$

$= 3.3\ln|6| - 2.4\ln|8| - 0.9\ln|2|$

$= 3.3(\ln|3| + \ln|2|) - 2.4\ln|2^3| - 0.9\ln|2|$

$= 3.3\ln|3| + 3.3\ln|2| - 7.2\ln|2| - 0.9\ln|2|$

$= 3.3\ln|3| - 4.8\ln|2|$ ①

(Total for Question 13 is 11 marks)

14. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4\sin 2y \qquad \frac{-\pi}{4} < y < \frac{\pi}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{dy}{dx}$ at the origin. **(2)**

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i). **(2)**

(c) Show that, for all points (x, y) lying on C,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found. **(3)**

a) $x = 4\sin 2y$

$\frac{dx}{dy} = 4(2\cos 2y)$ ①

$= 8\cos 2y$

Take reciprocal

$\frac{dy}{dx} = \frac{1}{8\cos 2y}$

At origin $(0,0)$ so sub $y=0$

$\frac{dy}{dx} = \frac{1}{8\cos(0)}$

$\cos(0) = 1$

$\frac{dy}{dx} = \frac{1}{8}$ ①

small angle approximation

bi) $\sin x \approx x$

$\sin 2y \approx 2y$ ①

$\therefore x = 4\sin 2y$

using $\sin 2y \approx 2y$

$x \approx 4(2y)$

$x \approx 8y$

bii) Value found in a) is the gradient of the line found in bi) ①

$y = \frac{1}{8}x$

can see by re-arranging that gradient same as value in a)

c) $\frac{dy}{dx} = \frac{1}{8\cos 2y}$

$x = 4\sin 2y$

$\sin^2 x + \cos^2 x = 1$

$\therefore \sin^2 2y + \cos^2 2y = 1$

$x^2 = 16\sin^2 2y$

using $\sin^2 2y = 1 - \cos^2 2y$ (1)

$x^2 = 16(1 - \cos^2 2y)$

$x^2 = 16 - 16\cos^2 2y$ (1)

$16\cos^2 2y = 16 - x^2$

$\cos^2 2y = 1 - \frac{x^2}{16}$

$\cos 2y = \sqrt{1 - \frac{x^2}{16}}$

$\frac{dy}{dx} = \frac{1}{8\sqrt{1 - \frac{x^2}{16}}} \begin{matrix} \times\sqrt{16} \\ \times\sqrt{16} \end{matrix}$

$= \frac{\cancel{\sqrt{16}}\ 4}{8\sqrt{16 - x^2}}$

$= \frac{1}{2\sqrt{16 - x^2}}$ (1)